Spherical Wave Sources in FDTD

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***Abstract*—A method is described which simulates the propagation of electromagnetic waves as spherical wave modes, approximated by an FDTD method. Modal equations in radius and time are discretized for explicit time-stepping. Angular functions are implemented analytically as required. Computed results for two examples are compared with analytic solutions—a resonatorand a dipole near a conducting sphere—to demonstrate the validity of the method with very good agreement. This method is intended, as a source condition in total/scattered FDTD methods, to allow for modeling of near-field object interactions without explicitly modeling the source.**

***Index Terms*—Difference equations, FDTD methods.**

I. Introduction

THE FDTD method has been used extensively in electromagnetic field modeling because of its ability to robustly handle interactions of fields with complex heterogeneous structures. In particular, the total/scattered field formulation has allowed for efficient implementation of arbitrarily directed uniform plane waves [1], consequently facilitating efficient modeling of far-field scattering problems. The total/scattered approach is not restricted to plane waves and can be expanded to any waveforms that can be easily described in analytical or semi-analytical form. For example, we have recently shown that an infinite line source can be implemented in a manner similar to plane waves [2].

While existing formulations of FDTD have been immensely successful, they are not well suited to problems that involve near field scattering/interaction problems, where both the source and object are in the same domain but at a substantial distance from each other. This is due to the exceedingly high demands for computational resources that may result from the domain size and/or dramatically different requirements for the mesh density in the source and object areas. Johnson and Rahmat-Samii [3] provide a solution for this problem by separating the domain into source and scatterer regions coupled by surface boundary radiation conditions. This method can incur large storage requirements for calculation of the radiation conditions.

Radiating sources can be accurately represented in the near-field by using spherical wave expansions (SWE) [4], [5]. For example, the SWE are often used to represent antennas measured on test ranges. Successful implementation of that idea within the FDTD framework would allow for the coupling between the source and scatterer to be accomplished by means of spherical modes and at a considerably lower cost than in the standard approach.

In the proposed method, a model of a source utilizing spherical waves is implemented in the total/scattered FDTD. Since transverse properties of spherical modes are known, the behavior of a mode can be represented on a one-dimesional (1-D) radial grid. Thus, much like the plane wave sources in the FDTD method, the spherical wave modes are time-stepped on 1-D staggered electric/magnetic field source grids in the radial direction, representing mode propagation in free space. Spherical wave modes can then be interpolated and summed on the Huygens' surface to represent the total field of the source, thus providing the coupling between the complex source and a scatterer using low cost 1-D grids. It is assumed that the object of interest is beyond the reactive near-field of the source and, therefore, there is no significant coupling between source and object. In this contribution, we outline a method and provide two numerical examples validating the technique.

TABLE I
Analytic Expressions for Modes

Field Component	TE	TM
H_r	$\frac{n(n+1)U}{r}$	0
H_θ	$\frac{1}{r}\frac{\partial}{\partial\theta}\frac{\partial}{\partial r}(rU)$	$\frac{\epsilon_0}{r\sin\theta}\frac{\partial}{\partial\varphi}\frac{\partial}{\partial t}(rV)$
H_φ	$\frac{1}{r\sin\theta}\frac{\partial}{\partial\varphi}\frac{\partial}{\partial r}(rU)$	$-\frac{\epsilon_0}{r}\frac{\partial}{\partial\theta}\frac{\partial}{\partial t}(rV)$
E_r	0	$\frac{n(n+1)V}{r}$
E_θ	$-\frac{\mu_0}{r\sin\theta}\frac{\partial}{\partial\varphi}\frac{\partial}{\partial t}(rU)$	$\frac{1}{r}\frac{\partial}{\partial\theta}\frac{\partial}{\partial r}(rV)$
E_φ	$\frac{\mu_0}{r}\frac{\partial}{\partial\theta}\frac{\partial}{\partial t}(rU)$	$\frac{1}{r\sin\theta}\frac{\partial}{\partial\varphi}\frac{\partial}{\partial r}(rV)$

II. Model Description

Consider Table I, describing spherical waves for both TE and TM modes, as translated from the frequency to the time domain [6]. All components are directly related to the functions U and V (where $\frac{U}{V} = \frac{a_{mn}u(r,t)}{b_{mn}v(r,t)} P_m^n(\cos\theta)e^{jm\varphi}$) through simple derivatives, with a_{mn} and b_{mn} being modal coefficients. For the Huygens' surface, polar and azimuthal derivatives can be determined analytically so, at this point, concern will be given to the time and radial derivatives. For the radial and time dependance of field components $f(r,t)$ (lowercase letters will be used henceforth for dependance on r and t only), consider a discretized scheme. In the TE modes, the field components (h_r, e_θ, e_φ) lie at locations $k\triangle r$ and (h_θ, h_φ) lie at $(k+1/2)\triangle r$. For TM modes, the field components (e_r, h_θ, h_φ) lie at locations $k\triangle r$

Manuscript received July 31, 2001; revised January 8, 2002. This work was supported by NSERC, while M. E. Potter was a student at the University of Victoria, Victoria, BC, Canada. The review of this letter was arranged by Associate Editor Dr. Ruediger Vahldieck.

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Publisher Item Identifier S 1531-1309(02)03971-5.

TABLE II
ANALYTIC AND COMPUTED RESONANT FREQUENCIES $(f_r)^{TM}_{m1p}$ (MHZ) FOR $n = 1$, FOR A SPHERICAL CAVITY OF RADIUS $r = 1$ m

p	1	2	3	4	5	6
Analytic	131.01	292.06	444.85	596.16	746.94	897.44
Computed	131	292	445	596	746	897

and $(e_\theta,\ e_\varphi)$ lie at $(k+1/2)\triangle r$. Magnetic field components are at timesteps $n\triangle t$ and electric field components at timesteps $(n+1/2)\triangle t$. For a particular source, the spherical wave coefficients and time excitation are incorporated as a hard source for each mode at the minimum radius. All modes are propagated on their own 1-D FDTD grid in radius and time. Each mode is then used as a source for points on the Huygens' surface in the total/scattered FDTD method, exactly analogous to a plane wave excitation, weighted by the appropriate polar and azimuthal functions.

Let us first consider the radial components. Letting u (or v) $= w/r$, we can formulate the wave equation in w [7]

$$\frac{\partial^2 w}{\partial r^2} - \frac{n(n+1)}{r^2}w - \frac{1}{c^2}\frac{\partial^2 w}{\partial t^2} = 0. \tag{1}$$

A difference equation corresponding to (1) is

$$w_k^{i+1} = \left(\frac{c\triangle t}{\triangle r}\right)^2 \left[w_{k+1}^i - 2w_k^i + w_{k-1}^i\right] - \left(\frac{c\triangle t}{r_k}\right)^2 n(n+1)w_k^i + 2w_k^i - w_k^{i-1}. \tag{2}$$

where the superscript i denotes the timestep index. This produces an FDTD method of calculating radial components. Examining Table I once again, we see that the other components can be determined directly from the radial components via simple radial or time derivatives. A stability criterion for this method has been reported in [8].

A. Radial Derivatives

The radial derivatives of concern, e.g., $h_\theta(r,t) = (1/r)(\partial/\partial r)[r^2 h_r(r,t)]$, are of a similar form. It needs to be emphasized that angular components are staggered from the radial components by half a grid cell and therefore they lend themselves immediately to a central difference representation

$$f_a(r,t) = \frac{1}{r}\frac{\partial}{\partial r}\left[r^2 f_r\right] = 2f_r + r\frac{\partial f_r}{\partial r} \Rightarrow$$
$$f_{a,k+1/2}^i = \left(f_{r,k+1}^i + f_{r,k}^i\right) + r_{k+1/2}\frac{\left(f_{r,k+1}^i - f_{r,k}^i\right)}{\triangle r} \tag{3}$$

where k is the radial index, a represents a given angular component, and f represents magnetic or electric fields.

B. Time Derivatives

Time derivatives, e.g., $e_\theta(r,t) = -\mu(\partial/\partial t) r h_r(r,t)$ are of a similar form also. Electric and magnetic fields are staggered in time by half a time step and the necessary components are co-located in radius. So, once again, it is a simple measure to represent the time derivative with a central difference scheme

$$g_a(r,t) = \rho\frac{\partial}{\partial t} r f_r \Rightarrow g_{a,k}^{i+1/2} = \rho r_k \frac{\left(f_{r,k}^{i+1} - f_{r,k}^i\right)}{\triangle t} \tag{4}$$

where $\rho \in \{\epsilon, \mu\}$ and f, g are electric or magnetic fields. This scheme requires the future value of the radial $f_{r,k}^{i+1}$ component. However, since the radial component depends only on previous radial components (and not on polar or azimuthal components), the i-th time step nonradial components can be evaluated after $f_{r,k}^{i+1}$ are already known, without loss of precision or any numerical overhead. Moreover, those components can be computed only in those locations where they are actually needed.

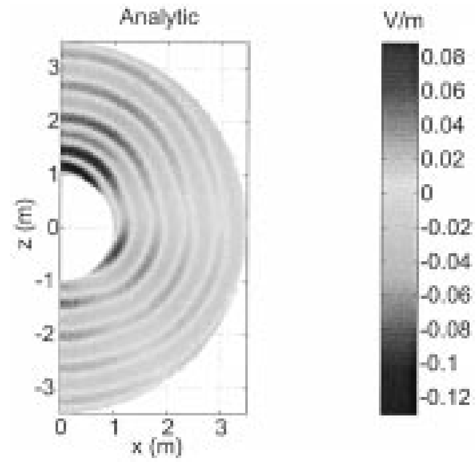

Fig. 1. Analytic E_θ fields for a dipole near a conducting sphere in V/m.

III. VALIDATION

A. Resonator

From [7], the resonant frequencies for the TM mode for a sphere of radius $r = 1$ m are tabulated in Table II for $n = 1$. Equation (2) was tested for a spherical cavity resonator of radius $r = 1$ m for this same mode, with $dr = 0.01$ m and $dt = 14$ ps. The cavity was excited with an impulse function and the simulation was allowed to proceed for 10 000 time-steps. To examine the results, the Fourier transform of the time signal at $r = 0.5$ m was taken at a frequency resolution of 1 MHz. The computed results are indicated in the second row of Table II, indicating excellent agreement with the analytic solutions.

B. Dipole and Conducting Sphere

As a second example, a much more demanding problem of a dipole near a conductive sphere is considered. For an x-directed electric dipole at $\overrightarrow{r}_0 = (r_0,\ 0,\ 0)$ above a perfectly conducting sphere (i.e., above the pole) with moment $(4\pi\epsilon_0/k)\widehat{x}$, the total fields are given in [9]. A number of cases were evaluated with results attaining similar precision. As an example case, a sphere of radius $a = 0.2$ m is reported here. The dipole was located 0.25 m from the sphere surface. The frequency of the radiated signal was chosen as $f = 0.5$ GHz, giving a wavelength $\lambda = 0.6$ m, or a wavenumber $k = (10\pi/3)$ m^{-1} $\simeq 10.47$ m^{-1}. The radial and time resolutions were $\triangle r = 0.01$ m and $\triangle t =$

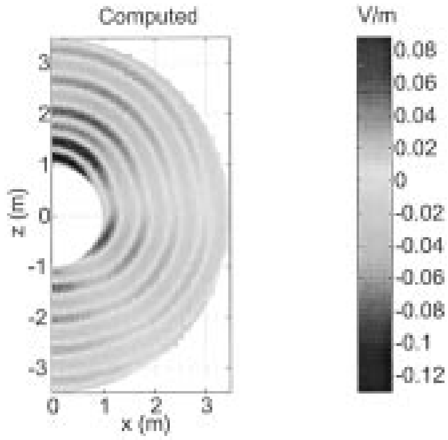

Fig. 2. Computed E_θ fields for a dipole near a conducting sphere in V/m.

14 ps, respectively, and 300 radial steps were used. Simulations were allowed to progress for 1000 time-steps. It is suggested in [4] that the expansion can be truncated at $N \simeq kr_0$ as the wavenumber/minimum sphere product. In this case, $kr_0 = 4.5$, so the expansion was truncated at $N = 7$ modes.

Analytic and computed results for E_θ are shown in Figs. 1 and 2 for $0 \leq \theta \leq \pi$ in $\pi/60$ increments, for $\varphi = 0$ (the xz-plane). Solutions are shown only for the last 200 radial steps, to avoid having large nearer-field wave amplitudes swamping out further-field amplitudes.

IV. Conclusion

A detailed method of propagating spherical waves via an FDTD approximation has been presented that should remove the necessity to explicitly model sources in near-field scattering simulations. Two validating examples have been reported representing excellent accuracy of the method. Work continues on applying this technique to study SAR and radiation patterns resulting from sources in close proximity to human body.

References

[1] A. Taflove, *Computational Electrodynamics—The Finite-Difference Time-Domain Method*. Boston, MA: Artech House, 1995.

[2] M. E. Potter, M. Okoniewski, and M. A. Stuchly, "Extension of FDTD method to nonuniform excitations," *Electron. Lett.*, vol. 34, no. 23, pp. 2216–2217, 1998.

[3] J. M. Johnson and Y. Rahmat-Samii, "MR/FDTD: A multiple-region finite-difference time-domain method," *Microw. Opt. Tech. Lett.*, vol. 14, no. 2, pp. 101–105, 1997.

[4] A. C. Ludwig, "Spherical-wave theory," in *The Handbook of Antenna Design*, A. W. Rudge, K. Milne, A. D. Oliver, and P. Knight, Eds. London, U.K.: Peregrinus, 1986, vol. 1 and 2, pp. 101–127.

[5] J. A. Stratton, *Electromagnetic Theory*. New York: McGraw-Hill, 1941, pp. 392–423.

[6] J. C. Slater and N. H. Frank, *Electromagnetism*. New York: McGraw-Hill, 1947, ch. XII.

[7] R. F. Harrington, *Time-Harmonic Electromagnetic Fields*. New York: McGraw-Hill, 1961.

[8] M. E. Potter and M. A. Okoniewski, "Stability criterion for radial wave equation in finite-difference time-domain method," *Electron. Lett.*, vol. 37, no. 8, pp. 488–489, 2001.

[9] J. J. Bowman, T. B. A. Senior, and P. L. E. Uslenghi, *Electromagnetic and Acoustic Scattering by Simple Shapes*. Amsterdam, The Netherlands: North-Holland, 1969.